Latin American Journal of Aquatic Research, 53(3): 518-527, 2025
DOI: 10.3856/vol53-issue3-fulltext-3198

*Research Article*

# The fishing Schaefer model with an adaptive effort law promoting equilibrium

**Fernando Córdova-Lepe**[1], **Rodrigo Gutiérrez**[1] & **Felipe N. Moreno-Gómez**[1]
[1]Universidad Católica del Maule, Talca, Chile
Corresponding author: Fernando Córdova-Lepe (fcordova@ucm.cl)

**ABSTRACT.** Based on the classical Schaefer model, which considers a given constant effort and defines a sustainable catch by the existence of an equilibrium abundance, this article addresses the problem of gradually changing the effort so that the present stock is at balance or is in a value near to balance. More precisely, a law of effort adaptation is proposed based on the equilibrium effort-population relationship of the classical model. The properties of the new effort-population balance are concluded, and comparisons are presented concerning the fishery's maximum sustainable yield, maximum economic yield, and net present value.

**Keywords:** Schaefer's model; dynamic fishery effort; adaptive effort; maximum sustainable yield; maximum economic yield

## INTRODUCTION

Fisheries have made significant contributions to achieving global food security and nutrition, with international efforts underway to meet the Sustainable Development Goals (FAO 2022). Ensuring the sustainable management of fisheries constitutes a key aspect that has been supported by both theoretical frameworks and reported real-world experiences (FAO 1997). Fishing effort is a well-known statistical parameter that gauges the stress on fish stocks. This indicator quantifies the level of fishing activity that occurs at a specific moment, often represented by the fish search duration or the amount of fishing gear of a specific type used by the fishing group involved over a given time period (FAO 2019). Fishing effort is defined on an ordinal or numerical scale, allowing comparisons of alternative deployment scenarios on the target species in a given fishing area.

The precautionary approach to fisheries management emphasizes the presence of high uncertainty in ecosystems and, therefore, encourages the implementation of conservative management procedures (FAO 1995). Following this, the primary objectives of regulatory entities should aim to maintain the equilibrium of fish abundance through regulations of fishing effort, where attaining a higher level of equilibrium is generally associated with a reduction in effort. Nevertheless, non-regulated fishing activities, guided by this approach, could be directed to operate at an optimal level of effort that maximizes productivity (maximum sustainable yield, MSY) or revenue (maximum economic yield, MEY) (Grafon et al. 2010, Narayanakumar 2017). However, it is important to note that the same level of yield can be achieved with different levels of effort due to the nonlinearity of the effort-production (sustainable) relationship (Smith 1969, Clark 1979).

To be feasible and viable, the implementation of solutions to the current global problem of marine overfishing (Walters & Martell 1944, Beverton 1998, Clark 2006) needs to consider several socioeconomic

Associate Editor: Sergio Neira

aspects. Therefore, to manage the impact of regulatory measures, the effort requires modifications at sustainable rates, allowing for the production of capture fisheries and the associated profits. In this sense, the introduction of an adaptive and continuous law to gradually adjust the effort, thereby achieving a stable population balance, is a necessity. Examples include the approach that considers feedback control of a given fishing effort (Matsuda & Abrams 2004) and the introduction of an effort that is dependent on target biomass (Ryzhenkov 2021).

Socially responsible management always aims to prevent marine resource overexploitation while guiding and promoting sustainable fishing activity (Kates et al. 2001, Córdova & Pinto 2002, Clark 2010). The Management Strategy Evaluation (MSE) assesses, through simulation, various potential harvesting control rules to achieve fishery objectives, which can be updated based on the observed results of the adopted strategy (Hordyk et al. 2021, FAO 2025). Another approach is Adaptive Fishery Management (AFM), which seeks to strike a balance between the exploitation and long-term conservation of abundance and ecosystem health by implementing an adaptive management effort. It is an open and practical management perspective, as it is flexible due to the capability to adapt as information and dynamics are understood (Walters 1986, Halbert 1993). In addition to monitoring, analyzing, and projecting the variables of interest, this adaptability also considers the involvement of different interested actors (e.g. fishermen, scientists, and regulatory authorities), thereby granting implementation legitimacy. Recall that much of what is evaluated and agreed by the AFM involves effort changes. Regardless of the meaning and value that scientific views have or incorporate in the concept of adaptation (McLain & Lee 1996), in the practice of any adaptive management, the decisions generally translate into variation in the deployed fishing effort.

In the mathematical modeling context, an adaptive effort is understood as an approach that transitions from considering effort as a parameter to a non-exogenous variable, in which its trajectory value responds to a dynamic rule. In this sense, Smith's model (Smith 1969), which explains the tragedy of commons (Hardin 1968), is an example of an adaptive effort, given that it is adapted to achieve utility increments. Models imposing control through closures constitute a way to implement adaptive changes in effort, as it is reduced within a time interval but spread over time (Córdova-Lepe et al. 2012, Castro-Santis et al. 2016).

In Reed (1991), an essay that presents and discusses the (perhaps) founding works of mathematical bioeconomics. Hotelling (1931), Gordon (1954), and Schaefer (1954) assured, "Indeed, it is only through the use of mathematical models that a rational discourse beyond the elementary level is possible on bioeconomic matters". At first glance, this statement appears exaggerated when the mathematical models mentioned by Gordon, Schaefer, and Hotelling are confronted with reality (data from specific fisheries), as general models do not adjust or adjust but with limited resolution. However, the value of these models lies in theorizing and providing a generic explanation of the essential patterns of the fishing phenomenon. These models have a strategic character that allows, through the aggregation of specificities, to advance to models with greater resolution possibilities, as would be the case when applying more tactical (e.g. control) or operational (e.g. projective) perspectives.

In the context of a fishery in progress and applying some constant effort level, we introduce a way to resolve an interesting challenge: determining an adaptive rule for the effort that ideally leads quickly, but in an economically sustainable way, toward an equilibrium stock. Furthermore, we ascertain properties of the ultimate levels of effort, abundance, and harvest toward which our proposed system converges. Specifically, we understand how these limit values of the variables depend on the initial state of the system. Methodologically, to address our research inquiry, we employ a simple mathematical model, adopting the foundational Gordon-Schaeffer theory as a baseline idealization. We envision a resource that lacks structural complexities, follows the logistic law of natural growth, and maintains proportionality (through the effort parameter) between harvest (capture per unit of effort, CPUE) and biomass. In particular, the effort rule proposed (the adaptive one) is modeled by an extra differential equation.

Finally, we declare an important motivation for this present work, an homage to our colleague and friend Eduardo González-Olivares, a mentor of many Latin American young researchers and one of the main introducers, through his articles, of the scientific area of the mathematical-bioeconomic perspective in Chile (González & Mena 1994, González-Olivares 1998, González-Olivares et al. 2009, Rojas-Palma & González-Olivares 2010, 2012).

## MATERIALS AND METHODS

Regarding the methodological aspect, we will proceed only through the epistemic approach of mathematical modeling. So, we will consider a fishing resource with sufficient conditions of uniformity and standard growth to be able to be modeled by a $(r, K)$-logistic equation.

In addition, with extraction conditions, i.e. fishing mortality, respecting the principle that the CPUE, a constant $E_0$, is proportional to the biomass:

$$N' = \mathcal{L}(N) - qNE_0 \quad \text{with} \quad \mathcal{L}(N) = rN(1 - N/K), \text{ and } N_0 = N(0) \tag{1}$$

where $\mathrm{N(t)}$ represents the population abundance at the instant $t, t \geq 0$, and the positive constant $q$ is a measure of the catchability of the resource.

Let us note that the Equation 1, rearranging its terms, admits the following equivalent form:

$$\mathrm{N}' = \mathrm{rN}\{\mathrm{N_e} - \mathrm{N}\}/\mathrm{K} \text{ with } \mathrm{N_e} = \mathrm{K}(1 - \mathrm{qE_0/r}) \tag{2}$$

Thus, at infinite time, we see that $N(t)$ tends to $N_e$ from any initial abundance $N_0$, if $\mathrm{E_0} < r/\mathrm{q}$, see Figure 1. That is, if the effort is limited, then $N_e$ is a globally stable positive equilibrium stock that decreases as $E_0$ increases; greater efforts imply extinction.

Reversing the reading of the fact that any constant effort defines a transition to a population in equilibrium, we have that, at any time, any population abundance $N$ is an equilibrium when the effort depends on the size of the resource, which will be denoted as $\bar{E}(N)$, given by:

$$\bar{E}(N) = r(1 - N/K)/q \tag{3}$$

that is, if we replace this value in Equation 1, we obtain $N' = 0$.

However, mobilizing impulsively and immediately the effort $E_0$ to this equilibrium effort $\bar{E}$ is not always possible in political-economic or simply technical terms. In this sense, we can assume that relative changes can only be made based on an evaluation (at each moment) of how distant, at any moment, the effort $E$ is from $\bar{E}$, that is, considering a law of evolution of the type:

$$\mathrm{E}' = \alpha\,\mathrm{E}\{\bar{\mathrm{E}}(\mathrm{N}) - \mathrm{E}\} \tag{4}$$

an expression that also assumes that the change is proportional ($\alpha$ is a scaling factor) to the value it presents. That is, it has a relative character.

Noting that $\mathrm{qN\bar{E}(N)} = \mathcal{L}(\mathrm{N})$, by replacing in (1), we get the differential system that follows:

$$\begin{cases} N' &= qN\{\bar{E}(N) - E\} \\ E' &= \alpha E\{\bar{E}(N) - E\} \end{cases} \tag{5}$$

defining a $\big(N(t), E(t)\big)$ the trajectory from any initial conditions $(\mathrm{N_0}, \mathrm{E_0})$.

The model defined by Equation 5 was developed under a generalist approach and, therefore, does not consider specific units. Thus, for future implementations that consider specific fisheries, it will be necessary to explicitly define abundance or stock size, time, and other parameter units.

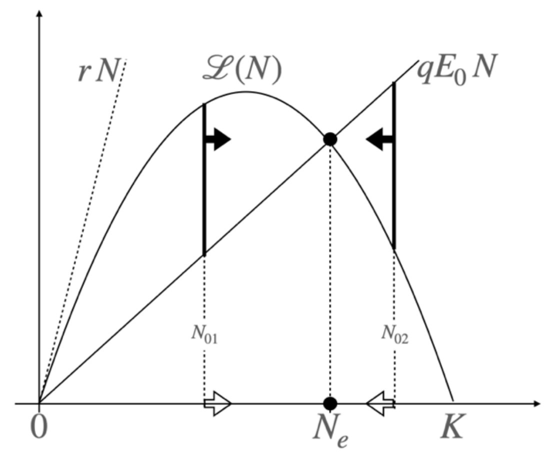


**Figure 1.** Representation of the natural growth $\mathcal{L}(\mathcal{N})$ and fishing mortality $\mathrm{qE_0N}$, as functions of population abundance (horizontal axis), determining by intersection a unique stable equilibrium $N_e$. That is, from any positive initial abundance as $N_{01}$ and $N_{02}$, the stock converges to $N_e$. Moreover, if the slope of the line $qEN$ is bigger than that of the tangent $rN$, that is, if $\mathrm{E_0} \geq \mathrm{r/q}$, then $N_e = 0$.

## RESULTS

Let us note that a direct deduction of the dynamic regime (Eq. 5) is that, except for proportionality constants, the change per individual of $N$ and the relative change of $E$, are equal, that is: $\mathrm{N'/N} = (\mathrm{q}/\alpha)(\mathrm{E'/E})$. This last equality, by direct integration, leads us to the relationship at any time:

$$(N/N_0)^{\alpha} = (E/E_0)^{q} \tag{6}$$

Note that this relationship leads us to the possibility of decoupling the variables and reducing (Eq. 5) to the scalar initial value problem:

$$N' = \mathcal{L}(N) - f(N),\ N(0) = N_0, \text{ where } f(N) = qE_0\,N(N/N_0)^{\alpha/q} \tag{7}$$

Now, about the geometry of the graph of $f(\cdot)$, as a function of $N$, we can say that $f(0) = 0$, $f(0) = 0$ (because $f(N)/N \to 0$ as $\mathrm{N} \to 0$),

$$f' = \frac{f}{N}\left(1 + \frac{\alpha}{q}\right),\ f'' = \frac{f}{N^2}\left(1 + \frac{\alpha}{q}\right)\frac{\alpha}{q} \text{ and } \left\{\frac{f}{N}\right\}' = \frac{\alpha}{q}\frac{f}{N^2}.$$

Then, the function is increasing, convex, and the angle $\tan^{-1}(f(N)/N)$ increases with $N$. In addition, an important property is that $N > N_0$ if and only if $f(N) > qE_0N$ since this implies that the system transits to a lower (or *resp*. bigger) equilibrium level $N_*$ of abundance in comparison with the constant one, i.e.

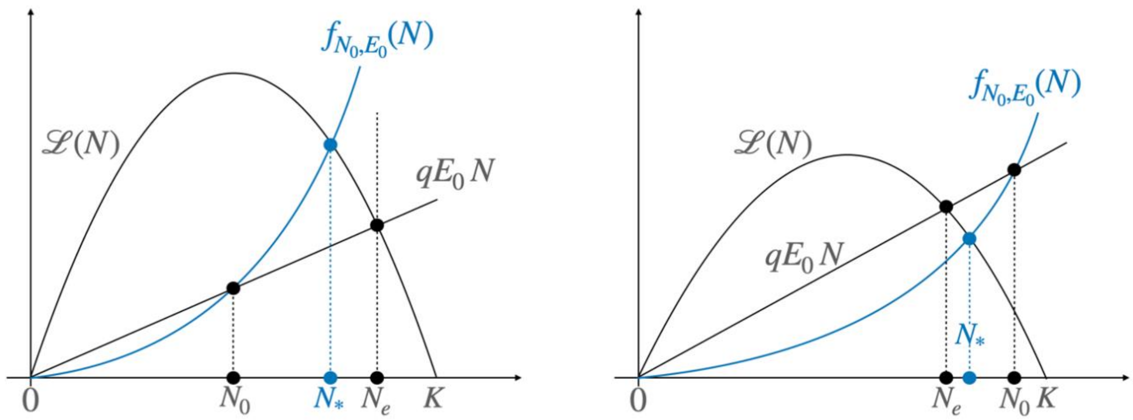


**Figure 2.** The equilibrium $N_*$, on the abundance axis (horizontal), always exists by the equalization of the natural growth $\mathcal{L}(N)$ with the fishing mortality given by $f(N)$ (in blue). Note that $N_*$ is always between the levels of abundance $N_e$ and $N_0$.

$N_* < N_e$ (or *resp.* $>$) when $N_0 < K$ (*resp.* $>$). See Figure 2.

### About equilibrium

To find a non-null equilibrium $(N_*, E_*)$ let us observe that $N' = 0$, if we have $\mathcal{L}(N) = f(N)$. This last equality has a unique solution: $N_* > 0$, which is dependent on $(N_0, E_0)$, and is represented by the blue point at the intersection of the curves in Figure 2. Then, from Equations 2 and 6, the respective effort's equilibrium is:

$$E_* = E_0\,(N_*/N_0)^{\alpha/q} = r(1 - N_*/K)/q \qquad (8)$$

Denoting $\mathcal{L}(\cdot) - f(\cdot)$, the right side of the differential equation in Equation 7, by $G(N)$We have that $\partial_N G$ is equal to $r(1 - 2N/K) - (f(N)/N)(\alpha + q)/q$. Then,

$$G'(N_*) = -\frac{r}{K}\left\{N_* + \frac{\alpha}{q}(K - N_*)\right\} < 0$$

which proves that the equilibrium $N_*$ is globally stable. Since $\partial_{N_*} E_* > 0$, that is, there is a positive correspondence between abundance equilibrium and effort equilibrium; it follows that the pair $(N_*, E_*)$ is a global equilibrium of the system (Eq. 5).

Noting that $\partial_{N_0} f(N) < 0$ and $\partial_{E_0} f(N) > 0$, it follows those larger initial stocks $N_0$ determine a larger equilibrium $N_*$. Similarly, larger initial efforts $E_0$ determine a smaller equilibrium $N_*$. Both assertions are visualized in Figures 3a-b, respectively.

### Estimating the equilibrium

Using the intersection of the tangent line to $f(N)$ at $(N_0, f(N_0))$ with the tangent line to the parabolic curve $\mathcal{L}(\mathcal{N})$ at $(N_e, L(N_e))$, we obtain an upper bound $N_*^+$ for $N_*$, see Figure 4. So, from the point in common between the tangent lines

$$L_f\colon\ y = f'(N_0)(N - N_0) + qE_0N_0 \text{ and } L_{\mathcal{L}}\colon\ y = \mathcal{L}'(N_e)(N - N_e) + qE_0N_e$$

we get

$$N_*^+ = \frac{(f'(N_0) - qE_0)N_0 + \big(qE_0 - \mathcal{L}'(N_e)\big)N_e}{f'(N_0) - \mathcal{L}'(N_e)}$$

Considering that $f'(N_0) = (\alpha + q)E_0$ and $\mathcal{L}'(N_e) = -r(1 - 2qE_0)$, replacing in $N_*^+$, we obtain

$$N_*^+ = \frac{\alpha E_0 N_0 + rK(1 - qE_0/r)^2}{(\alpha - q)E_0 + r}$$

Furthermore, using the intersection of the parabola with the line joining the origin with $(K, f(K))$, we have a lower bound given by

$$N_*^- = K\left\{1 - \frac{qE_0}{r}\left(\frac{K}{N_0}\right)^{\alpha/q}\right\}$$

under condition $qE_0/r < (N_0/K)^{\alpha/q}$.

### About maximum sustainable yield (MSY)

Let us note that the equilibrium production is given by $Y_* = f(N_*) = qE_*N_*$; this is

$$Y_* = r(1 - N_*/K)N_* = rqKE_*\big(1 - E_*/(r/q)\big), \quad (9)$$

where $N_*$ and $E_*$ depend on the starting condition $(N_0, E_0)$. Then, this yield is maximum if $(N_*, E_*)$ reaches the value $(N_{MSY}, E_{MSY}) \coloneqq \big(K/2, r/(2q)\big)$, determining $Y_* = Y_{MSY} \coloneqq rK/4$, as expected. Then, the question is which $E_0$, as a function of the other initial value $N_0$, determines $Y_* = Y_{MSY}$, the answer follows from $f_{E_0,N_0}(N_*) = Kr/4$, this is:

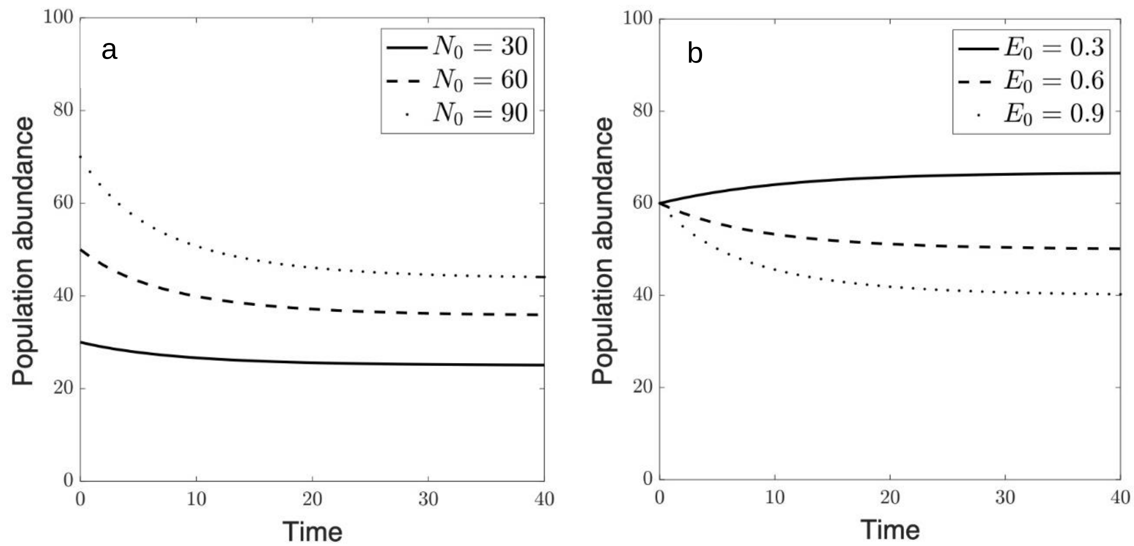


**Figure 3.** Temporal dynamics of stock according to model (Eq. 5) for different values of initial conditions, a) $N_0 \in \{30, 50, 70\}$ with $E_0 = 0.9$, and b) $E_0 \in \{0.3,\ 0.6,\ 0.9\}$ with $N_0 = 60$. The common parameters are $r = 0.1$, $K = 100$, $q = 0.1$, and $\alpha = 0.1$.

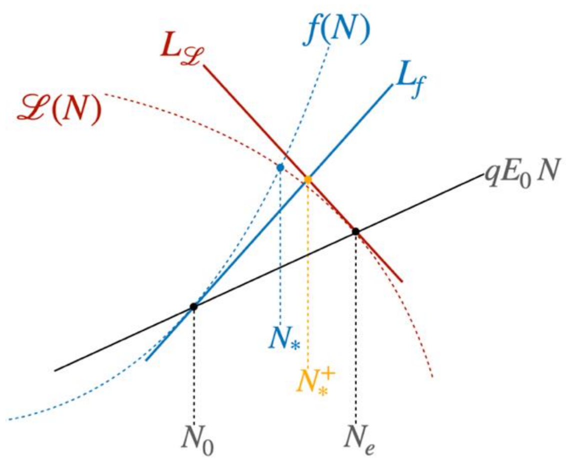


**Figure 4.** The intersection point between the respective tangents $L_{\mathcal{L}}(\cdot)$ and $L_f$ to the natural growth curve $\mathcal{L}$ in $\left(N_e, \mathcal{L}(N_e)\right)$ and fishing mortality $f(\cdot)$ in $\left(N_0, f(N_0)\right)$ stands out in yellow, which allows obtaining the upper bound $N_*^+$ at the value of $N_*$.

$$E_{0_{MSY}} = E_{MSY}\left(\frac{N_0}{N_{MSY}}\right)^{\alpha/q} = \frac{r}{2q}\left(\frac{N_0}{K/2}\right)^{\alpha/q} \quad (10)$$

which is a growing and concave (*resp*. convex) curve if $\alpha < q$ (*resp*. $\alpha > q$) and is shown, respectively, in the blue dashed (*resp*. dotted) line in Figure 5.

So, considering that. $E_0$ could be a parameter that admits a control margin; let us examine the sensitivity of $Y_*$ to said initial parameter. Using Equation 9 and

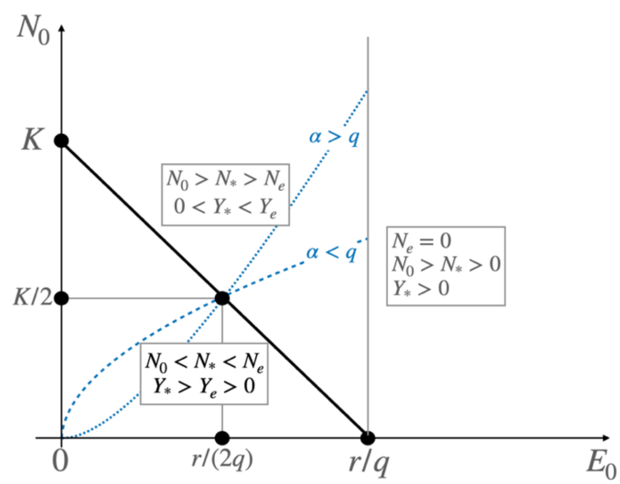


**Figure 5.** Comparison, on the map of initial abundance (vertical axis) and initial effort (horizontal axis) possibilities, of the equilibrium populations and the respective productions they determine between the case of constant effort and that of adaptive effort. In black, the plot of the line $N_0/K + E_0/(r/q) = 1$ to which the non-null equilibrium $(N_*, E_*)$ belongs. An initial condition on the blue line, dashed if $\alpha > q$ or dotted if $\alpha < q$, determines convergence to the central point $(N_{MSY}, E_{MSY}) = \left(K/2, r/(2q)\right)$.

$$\frac{1}{K}\frac{\partial N_*}{\partial E_0} + \frac{q}{r}\frac{\partial E_*}{\partial E_0} = 0,$$

we get $\frac{\partial Y_*}{\partial E_0} = 2q\frac{\partial E_*}{\partial E_0}\left\{N_* - \frac{K}{2}\right\} = qK\frac{\partial E_*}{\partial E_0}\left\{1 - \frac{E_*}{r/(2q)}\right\}$ (11)

Then, the first-order condition for an optimal $\partial_{E_0} Y_* = 0$ implies the equilibrium $(N_*, E_*)$ equal to $(N_{MSY}, E_{MSY}) := (K/2, r/(2q))$, and $Y_*$ equal to $Y_{MSY} = rK/4$, as expected.

Since $\frac{\partial E_*}{\partial E_0} = \frac{E_*/E_0}{1+(\alpha K/r)(E_*/N_*)} > 0$,

if $N_* > K/2$ (or equivalently $E_* < r/2q$), it follows $\partial_{E_0} Y_* > 0$. Symmetrically, if $N_* < K/2$ (or equivalently $E_* > r/2q$), then $\partial_{E_0} Y_* < 0$. Therefore, the point $(N_{MSY}, E_{MSY})$ marks the maximum sustainable yield when we have Equation10. See Figure 6.

**About maximum economic yield (MEY)**

Assuming a constant cost for each unit of effort deployed, we find that the cost function of the fishery (with adaptive effort) is $cE$ and tends to the value $C_* = cE_*$. Thus, the profit (utility) per unit of time at each instant, if a unit price $p$ per unit of capture is considered, is given by:

$$U = pY - cE \text{ where } Y = qEN,$$

an expression that, when the population-effort duo $(N, E)$ stabilizes, tends to $U_* = pY_* - cE_* = \{pqN_* - c\}E_*$, which will be positive if $N_* > c/(pq)$.

Notice that, from Equation 11, it is as follows.

$$\frac{\partial U_*}{\partial E_0} = \frac{\partial E_*}{\partial E_0}\left\{2pq\left(N_* - \frac{K}{2}\right) - c\right\}$$
$$= \frac{\partial E_*}{\partial E_0}\left\{pqK\left(1 - \frac{E_*}{r/(2q)}\right) - c\right\}$$

Then, $\partial_{E_0} U_* \geq 0$ if only if $N_* > N_{MEY}$, or equivalently $E_* < E_{MEY}$, with

$$N_{MEY} := \frac{K}{2}\left\{1 + \frac{c}{pqK}\right\} \text{ and } E_{MEY} := \frac{r}{2q}\left\{1 - \frac{c}{pqK}\right\}$$

As is known, we have

$$\frac{N_{MEY}}{N_{MSY}} > 1,\ \frac{E_{MEY}}{E_{MSY}} < 1 \text{ and } \frac{N_{MEY}}{N_{MSY}} + \frac{E_{MEY}}{E_{MSY}} = 2$$

Thus, if $(N_*, E_*)$ reaches the point $(N_{MEY}, E_{MEY})$, we have maximum profit, which occurs when the dependence on the initial effort $E_0$ respect $N_0$ is given by:

$$E_{0_{MEY}} = E_{MEY}\left(\frac{N_0}{N_{MEY}}\right)^{\alpha} =$$
$$E_{0_{MSY}}\left(1 - \frac{c}{pqK}\right)/\left\{1 + \frac{c}{pqK}\right\}^{\alpha/q} \qquad (12)$$

**Comparing net present values (NPV)**

It is interesting to compare the cases of fixed effort and adaptive one, the intertemporal economic profit in some determined time horizon $[0, T]$, or in perpetuity $[0, \infty]$. To accomplish this, let us consider the sum of the net present values of $U(t)$ assuming some discount rate $\delta > 0$:

$$NPV = \int_0^T \big(pY(t) - cE(t)\big)e^{-\delta t}dt$$

Figure 7, a 3×3 matrix of graphs, numerically compares the values of the $NPV$ of the adaptive model (Eq. 5) with the standard one (Eq. 1) for different combinations of price $p$ and cost $c$, and discount rates $\rho$ equals 5, 10, and 15% (firs, second and third columns respectively), with equality in all parameters shown in the Table 1.

**Table 1.** Common parameters for plots in Figure 7.

| $r$ | $K$ | $q$ | α | $N_0$ | $T$ |
|---|---|---|---|---|---|
| 0.9 | 100 | 0.1 | 0.2 | 60 | 30 |

As an initial effort, we consider the adaptive model $E_{0MSY}$ (Fig. 7, first matrix row) and EMSY for the standard model (Fig. 7, second matrix row).

We note that for any pair $(c, p)$, the *NVP* is greater than the corresponding standard, according to the comparison between the first and second rows of Figure 7, which is also reflected by their difference in the grayscale of Figure 7, third row. That is, given an initial stock $N_0$, a fishery with optimal constant effort $E_{MSY}$, can rent less if we compare it with the alternative of adaptive effort that begins with the effort given by $E_{0MSY}$, defined in Equation 10, and reaches, as a limit, the value $E_{MSY}$.

## DISCUSSION

Our approach corresponds to a strategic mathematical model, that is, a generalist type that combines explanatory capacity with technical simplicity (i.e. a model that, although simple in structure, is capable of capturing the essence of the phenomenon), which is in contrast with tactical and operational models aimed to represent specific fisheries, and therefore that should consider a higher resolution associated to those specific realities, allowing to obtain more accurate predictive potential. The economic approach proposed by M.B. Schaeffer is largely based on the formulation and analysis of Equation 1, serving as a clear example of a strategic model. Following this, when aiming to model and draw conclusions about particular fisheries and scenarios with increasing complexity, future versions of our model (Eq. 5) should consider enhancing the robustness of the variables of interest and their structural relationships. For instance, incorporating the idea of adaptive effort into the more robust and validated models of fisheries that the literature offers.

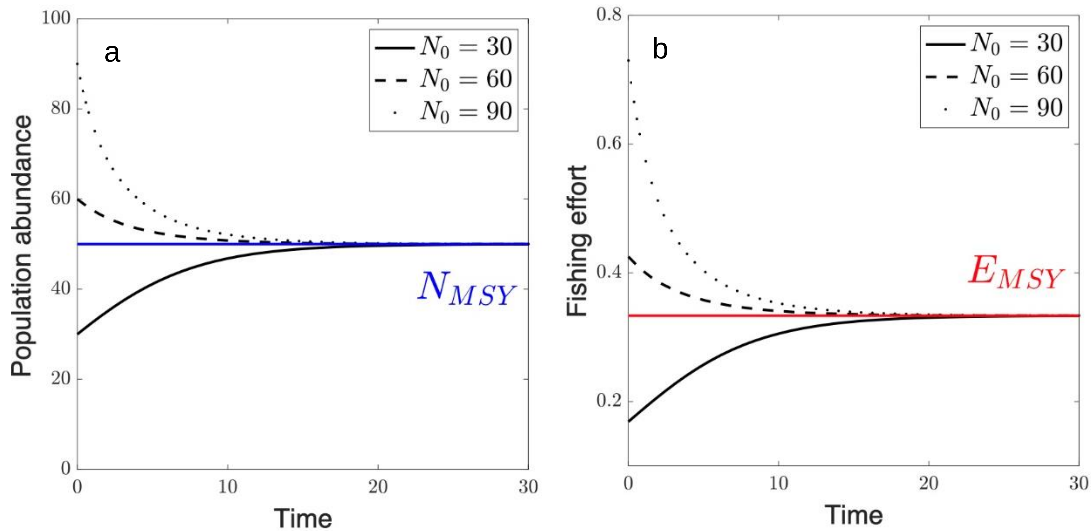


**Figure 6.** Temporal dynamics of the model (Eq. 5) for different initial conditions $(N_0, E_0)$, where $N_0 \in \{30, 50, 70\}$ and $E_0 = E_{0MSY}$ according to (10). Particularly, $E_0 \in \{0.1687, 0.4251, 0.7299\}$. The common parameters are $r = 0.2$, $K = 100$, $q = 0.3$, and $\alpha = 0.4$. In a), $N_{MSY} = 50$ and b) $E_{MSY} = 0.33$.

In line with this, a case of interest would be to examine the response of the adaptive effort model in the event of significant issues affecting fisheries, such as bycatch and/or discarding. As mentioned previously, this requires incorporating structural modifications into our model, which is a pending challenge for future studies.

We note that in the case of constant effort, if this is excessive, specifically $E_0 > r/q$, then the population of resources becomes extinct. However, in the case of adaptive (variable) effort, the system always transits to a limited effort that determines a non-zero equilibrium population $N_*$.

Regarding population levels, let us note that if $N_0 < N_e$ (*resp*. $>$), we find that the adaptive effort causes the population to grow (*resp*. decrease) towards equilibrium. However, this is less (*resp*. greater) than the constant stress case, that is, $N_* \in (N_0, N_e)$ (*resp*. $(N_e, N_0)$), see Figure 2. However, this is an aspect that is reversed if we observe what happens concerning sustainable production (in equilibrium) per unit of time. In effect, we have $Y_* := f(N_*) > Y_0 := qE_0N_e$ when $N_0 < N_e$, and conversely $Y_* < Y_0$ if $N_0 > N_e$. Moreover, for an initial stock $N_0$, there exists an initial effort given for Equation 10, which implies $Y_* = Y_{MSY} = rK/4$.

Considering some caveats, our results could be used for informed decision-making by scientific committees and sectoral authorities. The model could provide a first qualitative evaluation, adding information when establishing fishing efforts. Note that when considering the actual net value of a fishery that is dependent on the effort distribution through time, Figure 7 shows a comparison between a case of constant effort that tends to the maximum sustainable yield $E_{MEY}$. In general, for a greater portion of the parameter space (cost, price), the adaptive case with the initial effort given by Equation 10 is optimal in its present value; the exception is when the cost of the effort unit is relatively very low compared to the price, suggesting that in fishery regulation, under a situation of stable inputs and outputs, the limitation of the maximum effort should aim to the adaptive type governed by Equation 10.

An interesting future challenge, in the case of fisheries with regulated effort, whether by authorities or owners, is to mathematically explore (via strategic models as those exposed in this work) the dynamic expression of technically realistic effort trajectories that manage to combine the needs for economic utility and a responsible perspective. The last is on both the ecological and cultural aspects of the social environment.

a

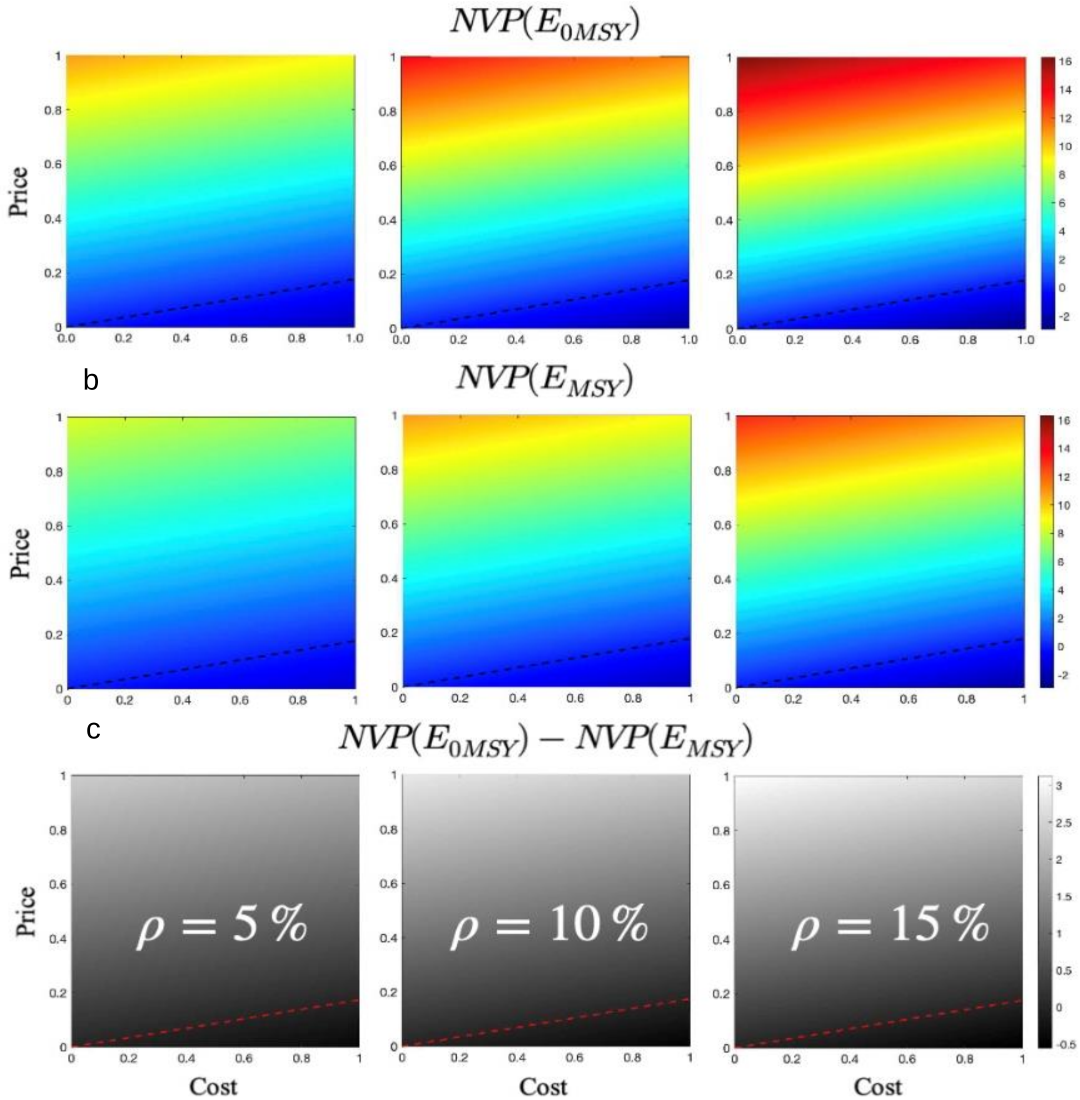


**Figure 7.** Net present values (NPV) associated with a) the model 1 (Eq. 1) and b) the model 5 (Eq. 5) for different parameter combinations of cost $c$ and price $p$, and taking $E_0 = E_{MSY}$ and $E_0 = E_{0MSY}$, respectively. The color scale represents the $NVP(E_{MSY})$ value. The common parameters are $r = 0.1$, $K = 100$, $q = 0.1$, and $\alpha = 0.2$, with $N_0 = 60$ and $\delta = 1$. In c), the NPV are compared. The black dashed line indicates $NVP(E_{MSY}) = 0$.

## Credit author contribution

F. Córdova-Lepe: conceptualization, methodology, formal analysis, writing-original draft, review and editing; R. Gutiérrez: numerical analysis, numerical plots, review and editing; F.N. Moreno-Gómez: ecological contextualization in introduction and conclusions sections, review and editing. All authors have read and accepted the published version of the manuscript.

## Conflict of interest

The authors declare no conflict of interest.

## REFERENCES


Beverton, R. 1998. Fish, fact, and fantasy: a long view. Reviews in Fish Biology and Fisheries, 8: 229-249.

Castro-Santis, R., Córdova-Lepe, F. & Chambio, W. 2016. An impulsive fishery model with environmental stochasticity. Feasibility. Mathematical Biosciences, 277: 71-76. doi: 10.1016/j.mbs.2016.04.001

Córdova, F. & Pinto, M. 2002. Bioeconomía matemática, explotación y preservación. CUBO, Matemática Educacional, 4: 49-64.

Córdova-Lepe, F., Del Valle, R. & Robledo, G. 2012. A pulse fishery model with closures as function of the catch: conditions for sustainability. Mathematical Biosciences, 239: 169-177. doi: 10.1016/j.mbs.2012.04.004

Clark, C.W. 1979. Mathematical models in the economics of renewable resources. SIAM Review, 21: 81-99. doi: 10.1137/1021006

Clark, C.W. 2006. The worldwide crisis in fisheries: economic models and human behavior. Cambridge University Press, New York.

Clark, C.W. 2010. Mathematical bioeconomics: The mathematics of conservation. Wiley Interscience, New York.

Food and Agriculture Organization (FAO). 1995. Precautionary approach to fisheries. Part 1: guidelines on the precautionary approach to capture fisheries and species introductions. FAO Fisheries Technical Paper N°350. FAO, Rome.

Food and Agriculture Organization (FAO). 1997. Fisheries management. FAO technical guidelines for responsible fisheries N°4. FAO, Rome.

Food and Agriculture Organization (FAO). 2019. FAO technical workshop on global harmonization of Tuna fisheries statistics. Rome, 19-22 March 2018. FAO Fisheries and Aquaculture Report N° 1239. FAO, Rome.

Food and Agriculture Organization (FAO). 2022. The state of world fisheries and aquaculture 2022. Towards blue transformation. FAO, Rome.

Food and Agriculture Organization (FAO). 2025. Management Strategy Evaluation (MSE). FAO, Rome. [https://www.fao.org/fishery/en/eaftool/eaf_tool_50/en]. Reviewed: March 27, 2025.

González, E. & Mena, J. 1994. Análisis cualitativo de un modelo de pesquerías de acceso abierto. Investigaciones Marinas, 22: 3-11.

González-Olivares, E. 1998. Una clase de modelos bioeconómicos continuos. Investigaciones Marinas, 26: 109-117.

González-Olivares, E., Aránguiz-Acuña, A., Ramos-Jiliberto, R., et al. 2009. Demographical analysis of the pink ling *Genypterus blacodes* (Schneider, 1801) in the austral demersal fishery: A matrix approach evaluating harvest and non-harvest states. Fisheries Research, 96: 216-222. doi: 10.1016/j.fishres.2008.11.006

Gordon, H.S. 1954. The economic theory of a common property resource: the fishery. Journal of Political Economy, 62: 124-142. doi: 10.1086/257497

Grafton, R.Q., Kompas, T., Chu, L., et al. 2010. Maximum economic yield. Australian Journal of Agricultural and Resource Economics, 54: 273-280. doi: 10.1111/j.1467-8489.2010.00492.x

Halbert, C.L. 1993. How adaptive is adaptive management? Implementing adaptive management in Washington State and British Columbia. Reviews in Fisheries Science, 1: 261-283. doi: 10.1080/10641269309388545

Hardin, G. 1968. The tragedy of the commons. Science, 162: 1243-1247.

Hordyk, A., Schirripa, M. & Rosa, D. 2021. Updates to the operating model uncertainty grid for the north Atlantic swordfish MSE. ICCAT, 78: 240-261.

Hotelling, H. 1931. The economics of exhaustible resources. Journal of Political Economy, 39: 137-175.

Kates, R.W., Clark, W.C., Corell, R., et al. 2001. Sustainability science. Science, 292: 641-642. doi: 10.1126/science.1059386

Matsuda, H. & Abrams, P.A. 2004. Effects of predator-prey interactions and adaptive change on sustainable yield. Canadian Journal of Fisheries and Aquatic Sciences, 61: 175-184. doi: 10.1139/f03-147

McLain, R.J. & Lee, R.G. 1996. Adaptive management: Promises and pitfalls. Environmental Management, 20: 437-448. doi: 10.1007/BF01474647

Narayanakumar, R. 2017. Maximum economic yield and its importance in fisheries management. In: Course manual summer school on advanced methods for fish stock assessment and fisheries management. Lecture Note Series N°2/2017. CMFRI, Kochi, pp. 252-257.

Reed, W.J. 1991. Discussion: bioeconomics. An essay on the classic papers. In: Scott, H., Milner, B.S. & Hotelling, H. (Eds.). Bulletin of Mathematical Biology, 53: 215-229.

Rojas-Palma, A. & González-Olivares, E. 2010. Regulation by optimal taxation of an open access single-species fishery considering Allee effect on renewable resource. In: Mondaini, R. (Ed.). BIOMAT 2009. World Scientific, Singapore, pp. 213-230.

Rojas-Palma, A. & González-Olivares, E. 2012. Optimal harvesting in a predator-prey model with Allee effect and sigmoid functional response. Applied Mathematical Modelling, 36: 1864-1874. doi: 10.1016/j.apm.2011.07.081

Ryzhenkov, A.V. 2021. Controlling effort, catch and biomass in models of the global marine fisheries. Journal of Physics: Conference Series, 2092: 012021. doi: 10.1088/1742-6596/2092/1/012021

Schaefer, M.B. 1954. Some aspects of the dynamics of population important to the management of the commercial marine fisheries. Inter-American Tropical Tuna Commission Bulletin, 1: 23-56.

Smith V.L. 1969. On models of commercial fishing. Journal of Political Economy, 77: 181-198.

*Received: December 26, 2024; Accepted: June 9, 2025*

Walters, C. 1986. Adaptive management of renewable resources. Macmillan, New York.

Walters, C.J. & Martell, S.J.D. 1944. Fisheries ecology and management. Princeton University Press, New Jersey.